WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 3 – January 27, 2010

Round 1: Similarity and Pythagorean Theorem

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. The sum of the areas of two similar triangles is 255. If the ratio of the lengths of their sides is 1:4, compute the area of the larger triangle.

2. Consider isosceles triangle ABC with $BC = AC$. Choose point P on side $\overline{AB}$, point R on side $\overline{BC}$ and point S on $\overline{AC}$ such that $\overline{PR} \perp \overline{BC}$ and $\overline{PS} \perp \overline{AC}$. If $PR = 6$, $PS = 8$, and $AB = 28$, compute the length of AP.

3. The hypotenuse of a right triangle is 3 more than the length of one leg and 6 more than the length of the other leg. Find the length of the triangle's median drawn to the longer leg of the triangle. Please express your answer in simple radical form.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. _____

(3 pts.) 3._____

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Round 2: Algebra 1 - Open

2

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

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1. Rene can complete a particular task after working for 8 days, while Bertrand can compete the same task in 12 days. How many days will it take to complete the task if both Rene and Bertrand work together?

2. How many gallons of a 70% antifreeze solution must be drained from a full 5 gallon radiator if the radiator is then re-filled with pure antifreeze making the new solution in the radiator 80% antifreeze?

3. From her house, Mary travels to New York at an average rate of 50 miles per hour and then returns home at an average rate of 32 mph. After taking 2 hours longer on the return trip than the time it took her to get to New York, she finds that she still has 17 miles to go before she arrives home. How many miles is it from New York to Mary's house?

ANSWERS

(1 pt.) 1._____ days

(2 pts.) 2._____ gallons

(3 pts.) 3._____ miles

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Round 3: Functions

3

All answers must be placed in the answer section at the bottom
NO CALCULATOR ALLOWED

1. Let $f(x) = 3x - 5$ and $g(x) = 4x + b$, for some constant b. Find the value of b such that $f(g(x)) = g(f(x))$.

2. Suppose $\dfrac{f(n+1)}{f(n)} = n$ for all positive integers n. If $f(1) = 1$, find the numerical value of $f(6)$.

3. Let $f(x) = x^2 - 1$, $g(x) = 2x - 3$ and $h(x) = f(g(x))$. If $x \geq \dfrac{3}{2}$, then h^{-1} is a function. Find the value of $h^{-1}(48)$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 4: Combinations and Permutations

4

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

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1. In how many ways can the letters in the "word" WOCOMAL be arranged if W is ***not*** in the first position?

2. Our math team is sponsoring the school dance. If our team consists of 10 boys and 10 girls, in how many ways can dance committees of 3 team members be formed if the committee must contain at least one boy and at least one girl?

3. The school district is having its professional development day for teachers. In how many ways can 5 math teachers and 7 science teachers be placed into two groups where no group contains more than 8 people?

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

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Round 5: Analytic Geometry

5

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Find the value of c so that the vertex of the parabola $y = x^2 - 8x + c$ lies on the x-axis.

2. Consider the circle whose equation is $x^2 - 12x + y^2 - 4y + 15 = 0$. Find the y-intercept of the line that contains a diameter of the circle and also passes through the point (3, 6).

3. Find the center of the circle that passes through the points $(-3,5)$, $(3,3)$ and $(11,19)$.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2. _____

(3 pts.) 3. (_____, _____)

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. Chicken nuggets are available in packages of 6, 9 and 20. What is the largest number of nuggets that you cannot buy when combining various packages?

2. Consider the polynomial equation $3x^3 + bx^2 - 6x + d = 0$. If one of the roots of the equation is $3 + i$, find the value of the sum $b + d$.

3. A function f has the property that $f(x+1) = 2f(x) + 1$. If $f(1) = 3$, find the value of $f(4)$.

4. Consider right triangle ABC with right angle $\angle A$. The bisector of angle C intersects side $\overline{AB}$ at point D. If $AD = 1$ and $BD = 3$, find the length of $\overline{BC}$ in simplest radical form.

5. Suppose you have a convex polygon with n sides. If the number of vertices of your polygon doubles, then the number of diagonals that the polygon had increases by y. The number y is always divisible by x, where $x > 1$. Find the numerical value of x.

6. If the graphs of $3x + 2y = 0$ and $3x^2 + 4y^2 = 12$ intersect at the point (a, b), compute the product ab.

7. When Dave is as old as Jan is now, the sum of their ages will be 36. The sum of their ages is now 28. How old is Dave now?

8. Three rugs have a combined area of 200 square meters. By overlapping the rugs to cover a floor area of 140 square meters, the total area covered by any two of the rugs is 24 square meters. What is the area of the floor (in square meters) that is covered where all three rugs overlap?

9. Let $A(2,2)$, $B(6,4)$, $C(12,1)$, $D(8,-2)$ and $E(4,-3)$. Determine the area of pentagon ABCDE.

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ANSWERS

Round 1

1. 240
2. 16
3. $3\sqrt{13}$ (only)

Round 2

1. $\frac{24}{5} = 4\frac{4}{5} = 4.8$
2. $\frac{5}{3} = 1\frac{2}{3} = 1.\overline{6}$
3. 225

Round 3

1. $-\frac{15}{2} = -7\frac{1}{2} = -7.5$
2. 120
3. 5

Round 4

1. 2160
2. 900
3. 1749

Round 5

1. 16
2. 10
3. (3, 13)

Team Round

1. 43
2. 48
3. 31
4. $3\sqrt{2}$ (only)
5. 3
6. $-\frac{3}{2} = -1\frac{1}{2} = -1.5$
7. 12
8. 18
9. 40